Competitive Information Disclosure in Random Search Markets*

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Abstract

We analyze the role of competition in information provision in random search markets. Multiple symmetric senders compete for the receiver's investment by disclosing information about their respective project qualities, and the receiver conducts random search to learn about the qualities of the projects. We show that in any symmetric pure strategy Nash equilibrium, each sender chooses a strategy with the lowest possible reservation value. There is no active search, and the receiver does not benefit from the competition of the senders.

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1 Introduction

We consider a model of competitive information disclosure in random search markets. Multiple symmetric senders, each of whom is endowed with a project, compete for the investment of a single receiver by disclosing information about their respective project qualities. The number of senders is small, and the receiver is assumed to observe the strategies of the senders. We study a search model where search is costly and the search process is assumed to be random.

Several motivations can be offered for considering this model:[1]

- First, even if the receiver is assumed to observe the chosen strategies of the senders, in many realistic settings, the receiver might have only limited ability to conduct directed search. For example, this could be due to the scheduling concerns on the part of the senders, which limits the extent to which the receiver performs a directed search.
- Second, an equivalent interpretation of our model is that the senders' strategies are not individually observable (which makes directed search impossible for the receiver), but the receiver has access to some "aggregate-level data" about the distribution of signals in the market, which guides the receiver's decision of whether to continue search. This is the case if the search process is via an intermediary that typically provides aggregate-level information of the competitive products in a given market.
- Third, we motivate our analysis from the following theoretical perspective. Most models assume either a random search process where the receiver does not observe each individual sender's strategy or a directed search process where the receiver perfectly observes each individual sender's strategy. The modeling choice makes a huge difference in terms of the results. For example, the main model in Au and Whitmeyer (2022) study the latter case and shows that the receiver potentially benefits from the competition of the senders, and the hidden information setting in Au and Whitmeyer (2022, Section 5.2) studies the former case and shows that there is no symmetric equilibrium in which consumers engage in active search.

[1] We are grateful to an anonymous referee for suggesting the second and third motivations below.

> The search setting of our model sits between the two polar cases explored in Au and Whitmeyer (2022) as we maintain the observability of each individual sender's strategy while restricting the receiver's ability to conduct directed search. Our finding highlights that it is the ability to conduct directed search, not simply the public observability of the signals, that drives a competitive market outcome.

Formally, we consider a model in which n symmetric senders commit to information disclosure mechanisms. The quality of each sender's project is either high or low. The common prior is that the qualities are independently and identically distributed. The receiver conducts random search, and incurs a search cost/ inspection cost to learn about the qualities of the senders' projects. Theorem 1 shows that in any symmetric equilibrium of this game, each sender chooses a strategy with the lowest possible reservation value. The receiver meets each sender i with equal probability $\frac{1}{n}$, and invests in his project regardless of the posterior. There is no active search, and the receiver does not benefit from the competition of the senders, as the receiver's expected payoff does not change when the number of senders increases.

We show that our result persists under various extensions to our basic model, when we incorporate a role for outside option, when each sender's project quality follows a general distribution, and when the senders are asymmetric. In all these extensions, we show that there is no active search, and the receiver does not benefit from the competition of the senders.

While our result is reminiscent of the classical Diamond paradox (Diamond (1971)), we emphasize that our model is different. Although our receiver conducts a random search, the receiver in our model perfectly observes each individual sender's strategy, including any deviations from the equilibrium (see Section 4.4 for a fuller discussion on this).

Our paper contributes to the strand of the information design literature that studies competitive information disclosure (with or without search frictions). Boleslavsky and Cotton (2018), Au and Kawai (2020), Au and Kawai (2021), and Hwang et al. (2019) analyze competitive information disclosure in settings with no search cost. Boleslavsky and Cotton (2018) analyze a Bayesian-persuasion game with two senders, using the observation that the incentive structure faced by a sender is similar to that of a bidder in an all-pay auction with complete information. Au and Kawai (2020) adopt an approach that builds upon the linear structure of payoff functions, which allows them to tackle the more general setting with multiple senders and study the effect of the number of senders on equilibrium disclosure policies.[2] They establish the unique symmetric equilibrium in this game. As the number of senders increases, each sender disclosures information more aggressively, and full disclosure by each sender arises in the limit of infinitely many senders. Au and Kawai (2021) study a model of competition in which two senders vie for the patronage of a receiver by disclosing information about the qualities of their respective proposals, which are positively correlated. Hwang et al. (2019) solve the competitive persuasion problem when the prior is absolutely continuous and allow firms to set prices as well. In contrast to these papers, we model a random search market with search frictions and show that the receiver does not benefit from the competition of the senders.

Au and Whitmeyer (2022) study competitive information disclosure by multiple senders with search frictions. In their model, the receiver conducts a directed search, and the main focus is the attraction motive. They characterize the unique symmetric equilibrium—the receiver potentially benefits from the competition of the senders. They further consider the case of hidden signals—the firms' signals are not directly observable to the consumer at the outset of her search—and show that the consumer does not find it worthwhile to actively search. In settings in which the senders choose the information disclosure and also set a price, Whitmeyer (2021) shows that there is no symmetric equilibria in which consumers engage in active search, if neither the signal nor the price is observable until a consumer incurs the search cost.

Board and Lu (2018) consider a search setting in which a receiver, at a positive search cost, sequentially samples senders who provide information concerning a common state. In contrast, in our setting, the senders have independent proposals, and they make disclosures simultaneously.

[2] Spiegler (2006) studies a closely related problem, although the motivation comes from the bounded rationality of the consumer.

2 The basic model

We start with the basic version of the model. The model here is kept simple, at some costs of realism, which will be addressed later.

There are n senders, each of whom is endowed with a project. They compete for the investment of a single receiver. The quality of sender i's project, denoted θ_i, is either high (H) or low (L), and is independently and identically distributed across the senders. The common prior is that each sender's project is of high quality with probability p. Each sender's objective is to maximize the probability that the receiver invests in his project. Without loss of generality, we normalize each sender's payoff to be 1 if the receiver invests in his project, and 0 otherwise. The receiver's valuation for a project is 1 if its quality is H, and 0 if its quality is L. The receiver invests in at most one project. In the baseline model, the receiver does not have an outside option, and the receiver always invests.

The timing of the game is as follows.

(1) At the beginning of the game, each sender i simultaneously commits to an information disclosure mechanism on the quality of his project, which consists of a message space M_i and a joint distribution on $\{H, L\} \times M_i$. It follows from standard Bayesian persuasion arguments (see Kamenica and Gentzkow (2011)) that each sender i chooses a distribution F_i on $[0, 1]$ with mean p. Let $\mathcal{F}$ denote the collection of all such distributions. The chosen information disclosure mechanisms are publicly posted.

(2) The receiver learns about the qualities of the senders' projects through random search. At each stage of the search, the receiver can stop her search and invest in a project of any visited sender. Alternatively, she can incur a search cost of c, visit an unvisited sender, and observe the signal realization. To avoid triviality, we assume that $c < p$.

We focus on symmetric pure strategy Nash equilibria in which all senders adopt the same strategy and the receiver adopts a tie-breaking rule that treats all senders identically.

3 Equilibrium analysis

3.1 Basics

For any $F \in \mathcal{F}$, let

$$H_F(z) = -c + \int_0^{z-} z \, \mathrm{d}F(x) + \int_z^1 x \, \mathrm{d}F(x).[3]$$

A solution to the equation $H_F(z) = z$ exists and is unique (see Weitzman (1979)). We denote the solution to this equation by v_F, and refer to it as the reservation value of F. It is convenient to use the following rearrangement of the equation $H_F(v_F) = v_F$:

$$c = \int_{v_F}^1 (x - v_F) \, \mathrm{d}F(x).$$

The reservation value plays an important role in our analysis. In particular, adopting similar arguments as in Weitzman (1979) to our setting, we have that for any strategy profile of the senders,

(1) the receiver should continue her search if every unvisited sender uses a strategy that has a weakly higher reservation value than the maximum sampled reward and at least one unvisited sender uses a strategy that has a strictly higher reservation value than the maximum sampled reward, and

(2) the receiver should stop search if every unvisited sender uses a strategy that has a weakly lower reservation value than the maximum sampled reward.

Let F_F denote the full disclosure strategy, that is,

$$F_F(x) = \begin{cases} 1 - p, & \text{if } x \in [0, 1); \\ 1, & \text{if } x = 1. \end{cases}$$

[3]Notation: we use $\int_a^b$ to denote the integral over the interval $[a, b]$, $\int_a^{b-}$ to denote the integral over the interval $[a, b)$, and $\int_{a+}^b$ to denote the integral over the interval $(a, b]$. We use $F(x)$ to denote the measure on the interval $[0, x]$, $F(x-)$ to denote the measure on the interval $[0, x)$, and $F(\{x\})$ to denote the measure of the point x.

Let F_N denote the null disclosure strategy, that is,

$$F_N(x) = \begin{cases} 0, & \text{if } x \in [0, p); \\ 1, & \text{if } x \in [p, 1]. \end{cases}$$

It is straightforward to calculate that $v_{F_F} = 1 - \frac{c}{p}$ and $v_{F_N} = p - c$. Below we collect some well-known properties of the reservation value, which will be used throughout our equilibrium analysis.

1. For any $F \in \mathcal{F}$, $0 < p - c \leq v_F \leq 1 - \frac{c}{p} < 1$.
2. $v_F = p - c$ if and only if $F(v_F-) = 0$.
3. $v_F = 1 - \frac{c}{p}$ if and only if $F = F_F$.

As a benchmark, first consider the case of a single sender. Since the receiver does not have an outside option, regardless of the sender's strategy, the receiver incurs the search cost c to meet the sender, and invests in his project. The receiver's expected payoff is $p - c$.

3.2 Equilibrium analysis

Theorem 1 shows that in any symmetric pure strategy Nash equilibrium (if it exists), each sender chooses a strategy with the lowest possible reservation value. Clearly, in any symmetric equilibrium, all senders get the same expected payoff of $\frac{1}{n}$.

Theorem 1. *Suppose that $F \in \mathcal{F}$. If $(F, F, \ldots, F)$ is a symmetric pure strategy Nash equilibrium, then*

$$v_F = p - c.$$

By Theorem 1, in any symmetric pure strategy Nash equilibrium $(F, F, \ldots, F)$, the receiver meets each sender i with equal probability $\frac{1}{n}$, and invests in his project regardless of the posterior q_i (since $v_F = p - c$, $F(v_F-) = 0$). The receiver's expected payoff is $p - c$, the same as her payoff when there is a single sender. In other words, there is no active search, and the receiver does not benefit from the competition of the senders.

Here, we provide a sketch of the proof for Theorem 1; the detailed proof can be found in Appendix B. Suppose that there exists a symmetric pure strategy Nash equilibrium $(F, F, \ldots, F)$ such that $v_F > p - c$. Step 1 - Step 4 first establish some properties that F necessarily satisfies:

Step 1. $F(v_F-) \neq 0$ (since $v_F > p - c$).

Step 2. F has no jumps on $(0, v_F)$.

Step 3. $F(0) = 0$.

Step 4. F^{n-1} is flat on $[x_F, v_F)$ for some $0 < x_F < v_F$.[4]

Step 1 is straightforward. For each of Steps 2 - 4, we include a figure depicting the payoff function induced by a violation of any of these steps for the case of two senders: Figure 1 illustrates that if F has a jump at some $z \in (0, v_F)$, then a sender could improve his payoff using a mean preserving spread around z; Figure 2 illustrates that if $F(0) > 0$, then a sender could improve his payoff using a mean preserving contraction around 0; Figure 3 illustrates that if F is linear on $[0, v_F)$, then a sender could improve his payoff using a mean preserving contraction around v_F. For Step 4, intuitively, F must put no measure just under v_F, because the search cost induces a discrete jump in the payoff function (as a function of the posterior) at the posterior v_F.

For any F that satisfies these properties, in Step 5, we show that sender 1 could deviate to another strategy F' with a slightly lower reservation value than v_F and places a larger measure on the interval $[v_F, 1]$ than F does. Such a deviation has two effects. On the one hand, if the receiver visits the other senders before sender 1, there is a smaller probability of eventually visiting sender 1 (since $v_{F'} < v_F$). On the other hand, if the receiver visits sender 1 and has a posterior on $[v_F, 1]$ (which has a higher probability under F'), then she will stop search. Since F must put no measure just under v_F (as shown in Step 4), we can construct an F' such that the second effect dominates the first one.

[4] The arguments for these steps parallel those in Au and Kawai (2020). In the setting with no search costs, they show that a strategy profile is an equilibrium if and only if the induced payoff functions (of posterior distributions) exhibit a particular linear structure, and they then use this structure to pin down the unique symmetric equilibrium. The detailed arguments differ from theirs as we consider search costs. For completeness, we include the detailed arguments for the first four steps.

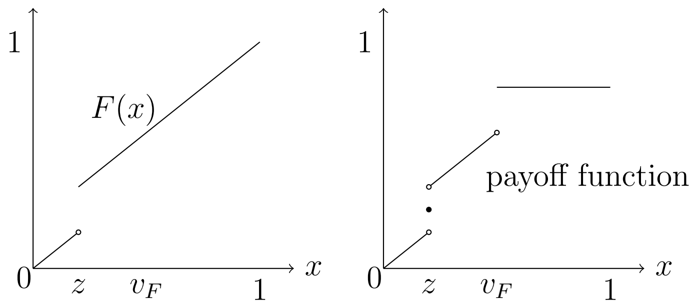

Figure 1: The figure on the left depicts an F that has a jump at some $z \in (0, v_F)$, and the figure on the right depicts the payoff function of sender 1. (for the case of two senders, for illustration purposes only).

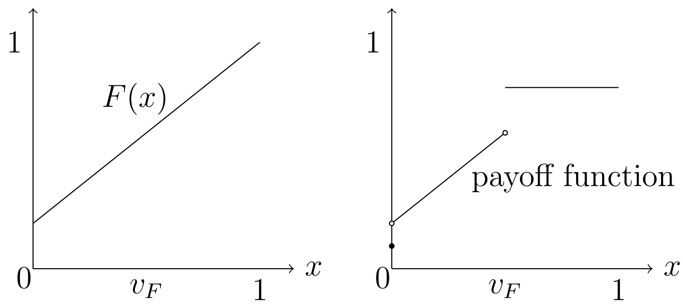

Figure 2: The figure on the left depicts an F with $F(0) = 0$, and the figure on the right depicts the payoff function of sender 1 (for the case of two senders, for illustration purposes only).

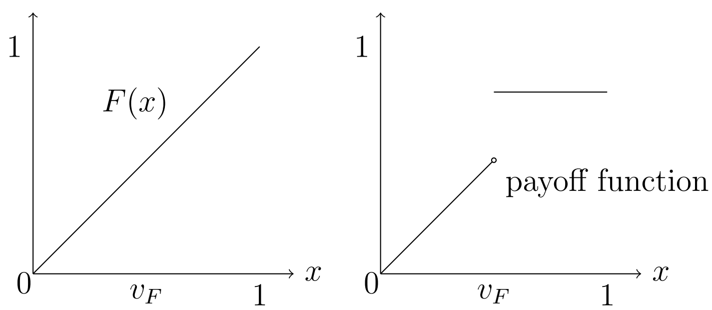

Figure 3: The figure on the left depicts an F that is linear on $[0, v_F)$, and the figure on the right depicts the payoff function of sender 1 (for the case of two senders, for illustration purposes only).

Theorem 1 shows that in any symmetric pure strategy Nash equilibrium $(F, F, \ldots, F)$ (if it exists), it must be that $v_F = p - c$. Having said that, we note that such an equilibrium might not exist. Proposition 1 below provides some necessary conditions and also some sufficient conditions for the existence of such an equilibrium. To pin down further necessary conditions, our analysis involves the search behavior of the receiver when one of the senders deviates to the following distribution

$$F_y(x) = \begin{cases} 1 - \frac{c}{1-y} - \frac{1}{y}(p - \frac{c}{1-y}) & \text{if } x \in [0, y); \\ 1 - \frac{c}{1-y} & \text{if } x \in [y, 1); \\ 1 & \text{if } x = 1. \end{cases}$$

for some $y \in [p - c, 1 - \frac{c}{p}]$ while the other senders continue to use F (with $v_F = p - c$). Suppose that there are $k - 1$ unvisited senders ($k \leq n$), where one of the senders uses F_y and the other $k - 2$ senders use F (with $v_F = p - c$). We show in Appendix A that there exists a cutoff $\psi_k(F_y)$ ($\psi_k(y)$ in short) such that the receiver continues her search if and only if the maximum sampled reward is less than $\psi_k(y)$.

Proposition 1. *Suppose that $F \in \mathcal{F}$ and $v_F = p - c$.*

(1) If $(F, F, \ldots, F)$ is a symmetric equilibrium, then

$$1 + \sum_{k=1}^{n-1} \Pi_{1 \leq j \leq k} F(\psi_{n-j+1}(x)) \leq \frac{x}{p - c}$$

for all $x \in [p - c, 1 - \frac{c}{p})$.

(2) If F satisfies the following:

$$1 + \sum_{k=1}^{n-1} F^k(x) \leq \frac{x}{p - c}$$

for all $x \in [p - c, 1 - \frac{c}{p})$, then $(F, F, \ldots, F)$ is a symmetric equilibrium.[5]

[5]When there are two senders, by working with the particular deviation strategy $F_y(x)$, we can show that the necessary conditions and sufficient conditions coincide (as $\psi_2(x) = x$). We proceed to work with this particular deviation strategy $F_y(x)$ in the case of more than two senders, but generally there is a gap.

The conditions in Proposition 1 provide some understanding of when a symmetric pure strategy Nash equilibrium does not exist.[6] Plainly, there are two sets of requirements on F, because F has to satisfy the conditions in Proposition 1 (which implies a lower bound on the mean) and also needs to be a feasible strategy (with mean p). Sometimes, such an F does not exist. For example, consider the setting with two senders and suppose that $p \leq 0.5$.[7] For $(F, F, \ldots, F)$ to be an equilibrium, we need

$$F(x) \leq \frac{x}{p-c} - 1$$

for all $x \in [p-c, 1-\frac{c}{p})$. But the mean of any such F is at least

$$\int_{p-c}^{2(p-c)} x \,\mathrm{d}(\frac{x}{p-c} - 1) = \frac{3}{2}(p-c).$$

Thus, for equilibrium existence, we would have to have that $\frac{3}{2}(p-c) \leq p$, or equivalently, $p \leq 3c$. If $p > 3c$, then a symmetric pure strategy Nash equilibrium does not exist. We provide a discussion for the case of n senders in Appendix C.

It is also clear from Proposition 1 that the conditions are more stringent and hence the lower bound of the mean of any distribution that satisfies these conditions gets larger when there are more senders.

Remark 1. (The degenerate distribution on the prior) The degenerate distribution on the prior p, even though it induces the minimal reservation value, is not necessarily an equilibrium. For a simple example, suppose that there are two senders, $p = 0.5$, and $c = 0.2$. The degenerate distribution on the prior induces the reservation value of 0.3. If both senders use the null information disclosure policy, then each sender gets an expected payoff of $\frac{1}{2}$. Now suppose sender 1 chooses the strategy that puts probability $\frac{11}{26}$ on $\{0\}$, probability $\frac{25}{156}$ on $\{0.52\}$, and probability $\frac{5}{12}$ on $\{1\}$. It is straightforward to calculate that the reservation value of this distribution is 0.52 and sender 1's payoff following this

[6] It is clear from Proposition 1 that the conditions are more demanding when there are more senders. We conjecture that given any parameter (p, c), there is a cutoff $\overline{n}$ such that when $n \leq \overline{n}$, a symmetric pure strategy Nash equilibrium exists in which there is no active search; when $n > \overline{n}$, only mixed strategy Nash equilibria exist in which there is active search and benefit of increased competition, paralleling the results in Au and Whitmeyer (2022).

[7] The calculation of the lower bound of the mean would be different if $p > 0.5$.

deviation is $\frac{15}{26}$. Hence, the new strategy is a profitable deviation for sender 1.

It is also easy to see that the degenerate distribution on the prior p violates the following necessary condition in Proposition 1:

$$1 + F(x) \leq \frac{x}{p - c}$$

for all $x \in [p - c, 1 - \frac{c}{p})$. For example, for the degenerate distribution on the prior p, for $x = 0.5$, $1 + F(0.5) = 2 > \frac{0.5}{0.5-0.2} = \frac{5}{3}$.[8]

4 Extensions

Theorem 1 shows that, in our basic model, there is no active search and the receiver does not benefit from the competition of the senders. In this section, we study various extensions to our basic model and show that our main result persists. Section 4.1 incorporates the role of an outside option. Section 4.2 considers the case of a general distribution. Section 4.3 analyzes the case of asymmetric senders. Lastly, in Section 4.4, we compare our model with the Diamond paradox.

4.1 Outside option

Our analysis can be readily extended to the case in which the receiver has an outside option u_0. Clearly,

(1) if $u_0 < p - c$, then our analysis in Section 3 remains unchanged; and

(2) if $u_0 > 1 - \frac{c}{p}$, then the receiver will not search at all.

In what follows, we consider the case in which $u_0 \in [p - c, 1 - \frac{c}{p}]$.

Suppose that there is only one sender. As in the Bayesian persuasion literature, we break ties in favor of the sender; that is, the receiver continues search if the reservation value of the sender's strategy equals u_0, and invests in the sender's project if the realized

[8] Also by Proposition 1, we can easily verify that the following is a Nash equilibrium: each sender uses the strategy that puts probability $\frac{1}{3}$ on 0.4, probability $\frac{1}{3}$ on 0.5, and probability $\frac{1}{3}$ on 0.6.

posterior is u_0. Without loss of generality, the sender uses a strategy with a reservation value that is at least u_0. It is easy to verify that, if the sender uses a strategy with a reservation value $v \geq u_0$, then the highest payoff of the sender is

$$\frac{c}{1-v} + \frac{p - \frac{c}{1-v}}{u_0},$$

which is decreasing in v, by using a strategy that places probability $\frac{c}{1-v}$ on 1, $\frac{p-\frac{c}{1-v}}{u_0}$ on u_0, and the remaining probability on 0. Thus, the sender would choose a strategy with the reservation value u_0. The receiver incurs the search cost c to meet the sender, and invests in his project if and only if the realized posterior is weakly higher than u_0. The receiver's expected payoff is u_0.

Now suppose that there are $n \geq 2$ senders. Using similar arguments as in the proof of Theorem 1, we can show that in any symmetric equilibrium $(F, F, \ldots, F)$, $v_F = u_0$. Thus, the receiver's expected payoff is u_0, the same as her payoff when there is a single sender. The receiver does not benefit from the competition of the senders.

4.2 General distribution

In the basic model, we assume that the quality of each sender's project takes only two values, high or low. Here, we consider the case in which the quality of each sender's project is drawn according to an atomless distribution function H with mean p and $\mathrm{supp}(H) = [0, 1]$ independently and identically across the senders. The receiver's payoff is θ if the receiver invests in a project with quality θ.

Following standard Bayesian persuasion arguments (see Kamenica and Gentzkow (2011)), each sender i chooses a distribution F_i that is a mean-preserving contraction (MPC) of H. Let $\mathcal{F}$ denote the collection of all such distributions. For any $F \in \mathcal{F}$, let v_F be such that

$$c = \int_{v_F}^{1} (x - v_F)\, \mathrm{d}F(x).$$

Clearly, $p - c \leq v_F \leq v_H$.

Theorem 2. *Suppose that $F \in \mathcal{F}$. If $(F, F, \ldots, F)$ is a symmetric equilibrium, then*

$$v_F = p - c.$$

By Theorem 2, in any symmetric equilibrium $(F, F, \ldots, F)$, the receiver meets each sender i with equal probability $\frac{1}{n}$, and invests in his project regardless of the posterior q_i (since $v_F = p - c$, we have $F(v_F-) = 0$.). There is no active search, and the receiver does not benefit from the competition of the senders.

4.3 Asymmetric senders

So far, our analysis focuses on the case of symmetric senders, and we show that there is no active search and the receiver does not benefit from the competition of the senders. The readers might wonder whether this still holds if the senders are asymmetric. In this section, we study the Nash equilibrium in a settings with two asymmetric senders.[9] There are two senders $i = 1, 2$. The quality of each sender i's project, θ_i, is either H or L. Sender 1's project is H with probability p_1, and sender 2's project is H with probability p_2. The receiver's valuation for a project is 1 if its quality is H, and 0 if its quality is L. Without loss of generality, we assume that $p_1 > p_2$.

To assess whether the receiver would benefit from the competition of the two senders, we would have to first establish a benchmark of the receiver's expected payoff in the case of a single sender. A natural benchmark to use is $\frac{p_1+p_2}{2} - c$. Intuitively, this is the receiver's expected payoff when there is a single sender, who is drawn from the two senders with equal probability.

Theorem 3. *(1) If (F_1, F_2) is a Nash equilibrium, then*

(a) $v_{F_2} = p_2 - c$,

(b) F_i concentrates on $[v_{F_j}, 1]$ with $i \neq j$, and

(c) $1 + F_2(x) \leq \frac{x}{p_1 - c}$ for all $x \in [p_1 - c, 1 - \frac{c}{p_1})$,
$2F_1(x) \leq \frac{x}{p_2 - c}$ for all $x \in [p_2 - c, v_{F_1})$, and

[9] The analysis of asymmetric senders for competitive information disclosure settings is in general a daunting question. For tractability, we focus on the case of two asymmetric senders.

$1 + F_1(x) \leq \frac{x}{p_2 - c}$ *for all* $x \in [v_{F_1}, 1 - \frac{c}{p_2})$.

(2) If (F_1, F_2) *satisfies the following:*

(a) $v_{F_2} = p_2 - c$,

(b) F_i *concentrates on* $[v_{F_j}, 1]$ *with* $i \neq j$, *and*

(c) $1 + F_2(x) \leq \frac{x}{p_1 - c}$ *for all* $x \in [p_1 - c, 1 - \frac{c}{p_1})$, *and*
$1 + F_1(x) \leq \frac{x}{p_2 - c}$ *for all* $x \in [p_2 - c, 1 - \frac{c}{p_2})$,

then (F_1, F_2) *is a Nash equilibrium.*

By Theorem 3, in any equilibrium (F_1, F_2), F_1 places no probability on the interval $[0, v_{F_2})$, and vice versa. Thus, the receiver meets each sender i with equal probability $\frac{1}{2}$, and invests in his project regardless of the posterior q_i. The payoff of each sender is $\frac{1}{2}$, and the receiver's expected payoff is $\frac{p_1+p_2}{2} - c$. There is no active search, and the receiver does not benefit from the competition of the senders.

Remark 2. (Equilibrium existence) As in the case of multiple symmetric senders, a pure strategy Nash equilibrium might not exist. Theorem 3 provides necessary conditions and also sufficient conditions for the existence of such an equilibrium. For example, Theorem 3 says that for (F_1, F_2) to be a Nash equilibrium, it must be that F_2 concentrates on $[v_{F_1}, 1]$. This cannot hold if, say, $p_1 - c > p_2$. In other words, the existence of the pure strategy equilibrium crucially relies on the perturbation from homogeneity being sufficiently mild.

4.4 Discussion and comparison with the Diamond paradox

In this subsection, we study a direct analogy of our model in the more familiar setting in which two firms compete by choosing prices. We show that in this setting, when the consumer perfectly observes each firm's price, including any deviation from the equilibrium (in line with the random search model considered in this paper), the monopoly outcome may not be an equilibrium.[10] This exercise highlights that our model is different from the Diamond paradox.

[10]We are grateful to the referees for suggesting this discussion.

Consider a simple setting with two firms, each selling a product that yield value 1 to the consumer, who has an outside option of value 0. The marginal cost of production for both firms is 0. The search cost is $c \in (0, \frac{1}{2})$, which is (following the literature) only incurred if the consumer visits the second firm (so visiting the first firm is free). The Diamond paradox result says that the unique equilibrium is for the firms to obtain the monopoly outcome: both charge a price of 1 and extract all of the surplus.

Now suppose that the prices posted by the firms are publicly observable, but the consumer can only conduct a random search. It is easy to see that the monopoly outcome is not an equilibrium. Indeed, suppose that the monopoly outcome is an equilibrium. On the equilibrium path, both firms get an expected payoff of $\frac{1}{2}$. Firm 1 could deviate to charging the price of $p_1 = 1 - c - \epsilon$ for some sufficiently small $\epsilon > 0$. As $\epsilon \to 0$, firm 1's expected payoff following the deviation converges to $1 - c > \frac{1}{2}$. This contradicts that the monopoly outcome is an equilibrium.

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A Search behavior of the receiver

In this section, we study the optimal search behavior of the receiver in the following scenario: there are $k-1$ unvisited senders ($k \leq n$), where one of the senders uses the following distribution

$$F_y(x) = \begin{cases} 1 - \frac{c}{1-y} - \frac{1}{y}(p - \frac{c}{1-y}) & \text{if } x \in [0, y); \\ 1 - \frac{c}{1-y} & \text{if } x \in [y, 1); \\ 1 & \text{if } x = 1. \end{cases}$$

for some $y \in [p-c, 1-\frac{c}{p}]$ while the other senders continue use the same strategy F with $v_F = p - c$.[11] Let x denote the maximum sample reward (where $x \geq p - c$). In this section, we pin down the search behavior of the receiver in this scenario. In particular, we show that there exists a cutoff $\psi_k(y)$ such that the receiver continues her search if and only if the maximum sampled reward is less than $\psi_k(y)$.

[11]It is straightforward to verify that $v_{F_y} = y$.

It follows from Weitzman (1979) that the receiver should stop search if the maximum sampled reward x is weakly higher than the reservation value of v_{F_y}. In what follows, we assume that $x < v_{F_y}$.

We proceed by induction. Suppose that $k = 2$. That is, there is only one unvisited sender, and this sender uses F_y. Let

$$V_2(x|F_y) = \left[1 - \frac{c}{1-y} - \frac{1}{y}(p - \frac{c}{1-y})\right] x + \left[\frac{c}{1-y} + \frac{1}{y}(p - \frac{c}{1-y})\right] y - c$$
$$= (1 - \frac{p-c}{y})x + p - c$$

denote the receiver's expected payoff conditional on continuing search and let

$$W_2(x|F_y) = \max\{x, V_2(x|F_y)\}$$

denote the receiver's maximum expected payoff when the receiver optimizes her search decision. That is, the receiver compares the value of x and $V_2(x|F_y)$ when deciding whether to continue search. Let $\psi_2(F_y) = y$. Since $V_2(\psi_2(F_y)|F_y) = \psi_2(F_y)$, $\psi_2(F_y) = y$ is the level of the maximum sampled reward such that the receiver is indifferent.

For $k = 3, 4, \ldots, n$, we define $V_k(x|F_y)$, $W_k(x|F_y)$, and $\psi_k(x)$ inductively. Let

$$V_k(x|F_y) = \frac{1}{k-1}\left[(1 - \frac{p-c}{y})x + p\right] + \frac{k-2}{k-1}\int_{p-c}^{1} W_{k-1}(\max\{x, z\}|F_y)\,\mathrm{d}F(z) - c$$

denote the receiver's expected payoff conditional on continuing search,[12] and let

$$W_k(x|F_y) = \max\{x, V_k(x|F_y)\}$$

denote the receiver's maximum expected payoff when the receiver optimizes her search decision. That is, the receiver compares the value of x and $V_k(x|F_y)$ when deciding

[12]There are two cases to consider. (1) The receiver visits the sender who uses F' with $\frac{1}{k-1}$ probability. Then the receiver stops search and gets a payoff of x if the posterior of F_y is 0 (recall that $x \geq p-c = v_F$), and the receiver stops search and gets a payoff of y (resp. 1) if the posterior of F_y is y (resp. 1) (recall that $y \geq p-c = v_F$). (2) With the remaining probability, the receiver visits one of the senders who use F.

whether to continue search. Let $\psi_k(F_y)$ be such that

$$\psi_k(F_y) = \frac{1}{k-1}\left[(1-\frac{p-c}{y})\psi_k(F_y) + p\right] + \frac{k-2}{k-1}\int_{p-c}^{1} W_{k-1}(\max\{\psi_k(F_y), z\}|F_y)\,\mathrm{d}F(z) - c.$$

Clearly, $V_k(\psi_k(F_y)|F_y) = \psi_k(F_y)$, and $\psi_k(F_y)$ is the level of the maximum sampled reward such that the receiver is indifferent. Lemma 1 below shows that $V_k(x|F_y)$ is continuous in x, and such a fixed point $\psi_k(F_y)$ exists and is unique.

Lemma 1. *(1) $V_k(x|F_y)$ is continuous in x, and a fixed point exists for $V_k(x|F_y)$.*

(2) $V_k(x|F_y) - x$ is strictly decreasing in x.

(3) There exists a unique $\psi_k(F_y)$ such that $V_k(\psi_k(F_y)|F_y) = \psi_k(F_y)$.

Proof. (3) follows immediately from (2). In what follows, we adopt an induction argument to show that (1) and (2) hold for any $k \leq n$. Recall that $V_2(x|F_y) = (1 - \frac{p-c}{y})x + p - c$. Thus, (1) and (2) are clearly true when $k = 2$.

Suppose that (1) and (2) are true for some $k = k'$. We process to show that (1) and (2) hold for $k = k' + 1$. Since $V_{k'}(x|F_y)$ is continuous in x, we have $W_{k'}(x|F_y)$ is also continuous in x, which further implies that $V_{k'+1}(x|F_y)$ is continuous in x. It is easy to calculate that

$$\begin{aligned}
V_{k'+1}(p-c|F_y) =& \frac{1}{k'}\left[(1-\frac{p-c}{y})(p-c) + p\right] + \frac{k'-1}{k'}\int_{p-c}^{1} W_{k'}(\max\{p-c, z\}|F_y)\,\mathrm{d}F(z) - c \\
>& \frac{1}{k'}\cdot p + \frac{k'-1}{k'}\int_{p-c}^{1} z\,\mathrm{d}F(z) - c \\
=& p - c
\end{aligned}$$

and

$$\begin{aligned}
V_{k'+1}(1|F_y) =& \frac{1}{k'}\left[(1-\frac{p-c}{y})\cdot 1 + p\right] + \frac{k'-1}{k'}\int_{p-c}^{1} W_{k'}(\max\{1, z\}|F_y)\,\mathrm{d}F(z) - c \\
\leq& \frac{1}{k'}\cdot 1 + \frac{k'-1}{k'}\int_{p-c}^{1} 1\,\mathrm{d}F(z) - c \\
=& 1 - c
\end{aligned}$$

$<1.$

Thus, there exists some $x^* \in (p-c, 1)$ such that $V_{k+1}(x^*|F_y) = x^*$. This proves (1) for $k = k' + 1$.

For $x' > x$, we have

$$\begin{aligned}
&V_{k'+1}(x'|F_y) - V_{k'+1}(x|F_y) \\
=&\frac{1}{k'}(1-\frac{p-c}{y})(x'-x) + \frac{k'-1}{k'}\left[\int_{p-c}^{1} W_{k'}(\max\{x',z\}|F_y)\,\mathrm{d}F(z) - \int_{p-c}^{1} W_{k'}(\max\{x,z\}|F_y)\,\mathrm{d}F(z)\right] \\
<&\frac{1}{k'}(x'-x) + \frac{k'-1}{k'}\left[(W_{k'}(x'|F_y) - W_{k'}(x|F_y))F(x) + \int_{x}^{x'} (W_{k'}(x'|F_y) - W_{k'}(z|F_y))\,\mathrm{d}F(z)\right] \\
\leq&\frac{1}{k'}(x'-x) + \frac{k'-1}{k'}(x'-x)F(x') \\
\leq&x'-x,
\end{aligned}$$

where the second inequality holds since for any $z \in [x, x']$,

$$W_{k'}(x'|F_y) - W_{k'}(z|F_y) = \max\{x', V_{k'}(x'|F_y)\} - \max\{z, V_{k'}(z|F_y)\} \leq x' - z \leq x' - x.$$

This proves (3) for $k' = k + 1$. □

B Proofs of Theorem 1 and Proposition 1

We first consider the case in which $v_F > p - c$, and show that there is no symmetric pure strategy Nash equilibrium in this case (Theorem 1).[13] We then consider the case in which $v_F = p - c$ (Proposition 1).

Case I: $p - c < v_F \leq 1 - \frac{c}{p}$. Step 1 - Step 4 below establish properties that F necessarily satisfies. Step 5 shows that sender 1 has a profitable deviation, which

[13]In the case of two senders, since the two senders play a zero-sum game, this further implies that there is no (symmetric or asymmetric) equilibrium in which the reservation value of some sender's strategy is greater than $p-c$. To see this, suppose that (F_1, F_2) is a Nash equilibrium where $v_{F_i} > p-c$ for some i. By symmetry, (F_2, F_1) is also a Nash equilibrium. The inter-changeability property of zero-sum games (see Osborne and Rubinstein (1994, Proposition 22.2)) implies that (F_i, F_i) is a Nash equilibria.

contradicts that $(F, F, \dots, F)$ is a Nash equilibrium.

Step 1. Since $v_F > p - c$, $F(v_F-) \neq 0$.

Step 2. F *has no jumps on* $(0, v_F)$.

Suppose to the contrary, F has a jump at some $z \in (0, v_F)$. We show that there exists a profitable deviation for sender 1, which contradicts that $(F, F, \dots, F)$ is a Nash equilibrium.

Let $G = G_1 + G_2$ where

1. G_1 is a finite measure with total measure $1 - F(\{z\})$ such that $G_1(A) = F(A)$ for any $A \subseteq [0, z) \cup (z, 1]$, and
2. G_2 is a finite measure with total measure $F(\{z\})$ such that

$$G_2(\{z - n\epsilon\}) = \frac{1}{n+1} F(\{z\}) \text{ and } G_2(\{z + \epsilon\}) = \frac{n}{n+1} F(\{z\}),$$

   where $\epsilon > 0$ is sufficiently small such that $0 < z - n\epsilon < z < z + \epsilon < v_F$.

Clearly, G is a probability measure, and has the same mean and reservation value as F.

The difference of sender 1's expected payoff under G and F is at least (in the first line below we break ties against sender 1 for the calculation of sender 1's payoff at the two posteriors $z - n\epsilon$ and $z + \epsilon$)

$$\begin{aligned} &\frac{1}{n+1} F(\{z\}) F^{n-1}((z - n\epsilon)-) + \frac{n}{n+1} F(\{z\}) F^{n-1}((z + \epsilon)-) \\ &- F(\{z\}) \sum_{0 \leq j \leq n-1} \left[\frac{1}{j+1} \frac{(n-1)!}{j!(n-j-1)!} F^{n-j-1}(z-) F^j(\{z\}) \right], \end{aligned}$$

which converges to

$$\begin{aligned} & F(\{z\}) \Big[\frac{1}{n+1} F^{n-1}(z-) + \frac{n}{n+1} F^{n-1}(z) - \frac{1}{n} \frac{F^n(z) - F^n(z-)}{F(z) - F(z-)} \Big] \\ = {} & F(\{z\}) \Big[\frac{1}{n+1} F^{n-1}(z-) + \frac{n}{n+1} F^{n-1}(z) - \frac{1}{n} \sum_{0 \leq j \leq n-1} F^{n-j-1}(z) F^j(z-) \Big] \\ \geq {} & \frac{1}{n(n+1)} F(\{z\}) \Big[F^{n-1}(z) - F^{n-1}(z-) \Big] > 0 \end{aligned}$$

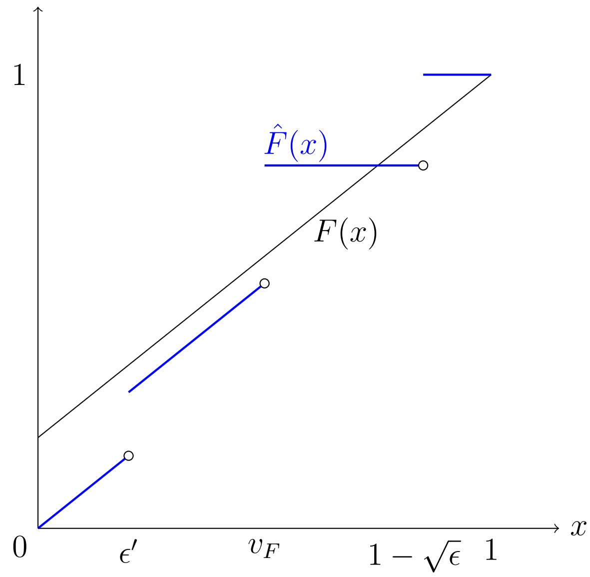

Figure 4: F and $\hat{F}$ in Step 3 of the proof (for illustration purposes only).

as $\epsilon \to 0$.

Step 3. $F(0) = 0$.

Suppose to the contrary, $F(0) > 0$. We show that there exists a profitable deviation for sender 1. For sufficiently small $\epsilon > 0$, let χ and ϵ' be such that

$$\chi = F(0) + \frac{c}{1-v_F} - \frac{c}{1-\epsilon-v_F} \text{ and } \chi \cdot \epsilon' + \frac{c}{1-\epsilon-v_F}(1-\sqrt{\epsilon}) = \frac{c}{1-v_F}.$$

Since $\epsilon' \to 0$ as $\epsilon \to 0$, we can choose ϵ such that $0 < \chi < F(0)$ and $0 < \epsilon' < v_F < 1-\sqrt{\epsilon} < 1$. Consider the following distribution $\hat{F}$ (see Figure 4):

$$\hat{F}(x) = \begin{cases} F(x) - F(0), & \text{if } x \in [0, \epsilon'); \\ F(x) - F(0) + \chi, & \text{if } x \in [\epsilon', v_F); \\ 1 - \frac{c}{1-v_F} - F(0) + \chi = 1 - \frac{c}{1-\epsilon-v_F}, & \text{if } x \in [v_F, 1-\sqrt{\epsilon}); \\ 1, & \text{if } x \in [1-\sqrt{\epsilon}, 1]. \end{cases}$$

$\hat{F}$ and F have the same mean, since

$$
\begin{aligned}
&\int_0^1 x \,\mathrm{d}\hat{F}(x) - \int_0^1 x \,\mathrm{d}F(x) \\
&= \chi \cdot \epsilon' + \big(1 - \frac{c}{1-v_F} - F(v_F-)\big) \cdot v_F + \frac{c}{1-\epsilon-v_F} \cdot (1-\sqrt{\epsilon}) - \int_{v_F}^1 x \,\mathrm{d}F(x) \\
&= \frac{c}{1-v_F} + \big(1 - \frac{c}{1-v_F} - F(v_F-)\big) \cdot v_F - \int_{v_F}^1 x \,\mathrm{d}F(x) \\
&= 0,
\end{aligned}
$$

where the last line uses the definition of the reservation value.

Since

$$\int_{v_F}^1 (x - v_F) \,\mathrm{d}\hat{F}(x) - c = \frac{c}{1-\epsilon-v_F} \cdot (1 - \sqrt{\epsilon} - v_F) - c < 0$$

and

$$\int_{v_F+\epsilon-\sqrt{\epsilon}}^1 \big(x - (v_F + \epsilon - \sqrt{\epsilon})\big) \,\mathrm{d}\hat{F}(x) - c = \int_{v_F+\epsilon-\sqrt{\epsilon}}^{v_F} \big(x - (v_F + \epsilon - \sqrt{\epsilon})\big) \,\mathrm{d}\hat{F}(x) > 0,$$

we have $v_F + \epsilon - \sqrt{\epsilon} < v_{\hat{F}} < v_F$. Thus, as $\epsilon \to 0$, $v_{\hat{F}} \to v_F$ and $F(v_{\hat{F}}) \to F(v_F-)$. Furthermore, $0 < \epsilon' < v_{\hat{F}} < v_F < 1 - \sqrt{\epsilon} < 1$ for $\epsilon > 0$ sufficiently small.

The difference of sender 1's expected payoff under $\hat{F}$ and F is at least

$$
\begin{aligned}
&\chi \cdot F^{n-1}(\epsilon') - F(0) \cdot \frac{1}{n} F^{n-1}(0) \\
&+ \int_{v_{\hat{F}}}^{v_F-} \frac{1}{n} F^{n-1}(x) \,\mathrm{d}\hat{F}(x) - \int_{v_{\hat{F}}}^{v_F-} F^{n-1}(x) \,\mathrm{d}F(x) \\
&+ \big(1 - \hat{F}(v_F-)\big) \frac{1}{n} \sum_{k=0}^{n-1} F^k(v_{\hat{F}}) - \big(1 - F(v_F-)\big) \frac{1}{n} \sum_{k=0}^{n-1} F^k(v_F-),
\end{aligned}
$$

since

(1) in the first term of the second line, for the calculation of sender 1's payoff at any posterior $q_1 \in [v_{\hat{F}}, v_F-)$, we only include the scenario in which the receiver visits sender 1 first (which happens with probability $\frac{1}{n}$) and all senders other than sender 1 have posteriors less than q_1, and

(2) in the first term of the third line, for the calculation of sender 1's payoff at any posterior $q_1 \in [v_F, 1]$, we only include the scenarios in which all senders other than sender 1 have posteriors less than $v_{\hat{F}}$.

This lower bound converges to $\frac{n-1}{n} F^n(0) > 0$ as $\epsilon \to 0$. Thus, sender 1 has a profitable deviation.

Step 4. F^{n-1} is linear on $[0, x_F]$ for some $0 < x_F < v_F$ and flat on $[x_F, v_F)$.

Since $F(0) = 0$ and F has no jumps on $(0, v_F)$, the payoff of sender 1 at any posterior $q_1 \in [0, v_F)$ is $F^{n-1}(q_1)$. Thus, F^{n-1} has to be linear on $[0, x_F)$ for some $0 < x_F \leq v_F$ and flat on $[x_F, v_F)$. Otherwise, sender 1 could do a mean-preserving spread or a mean-preserving contraction on $[0, v_F)$ without changing the reservation value to obtain a higher payoff. Next, we show that $x_F < v_F$. Since sender 1's payoff at the posterior v_F is $\frac{1}{n} \sum_{k=0}^{n-1} F^k(v_F-) > F^{n-1}(v_F-)$, the payoff of sender 1 has a jump at the posterior v_F. Thus, if F^{n-1} is linear on $[0, v_F)$, sender 1 could do a mean-preserving spread on $[0, v_F]$ without changing the reservation value to obtain a higher payoff.

Step 5. Sender 1 has a profitable deviation.

Let q denote the slope of F^{n-1} on $[0, x_F]$, and let $\hat{q} = q^{\frac{1}{n-1}}$. Consider the following strategy F' (see Figure 5):

$$F'(x) = \begin{cases} 0, & \text{if } x \in [0, y); \\ \hat{q}(x-y)^{\frac{1}{n-1}}, & \text{if } x \in [y, x_F); \\ \hat{q}(x_F - y)^{\frac{1}{n-1}}, & \text{if } x \in [x_F, v_F); \\ \hat{q}(x_F - y)^{\frac{1}{n-1}} + \kappa, & \text{if } x \in [v_F, 1); \\ 1, & \text{if } x = 1, \end{cases}$$

where $y > 0$ is sufficiently small, and

$$\kappa = \frac{\frac{\hat{q}}{n}(x_F - y)^{\frac{n}{n-1}} + \hat{q}y(x_F - y)^{\frac{1}{n-1}} + 1 - \hat{q}(x_F - y)^{\frac{1}{n-1}} - p}{1 - v_F}$$

such that the mean of F' is p.

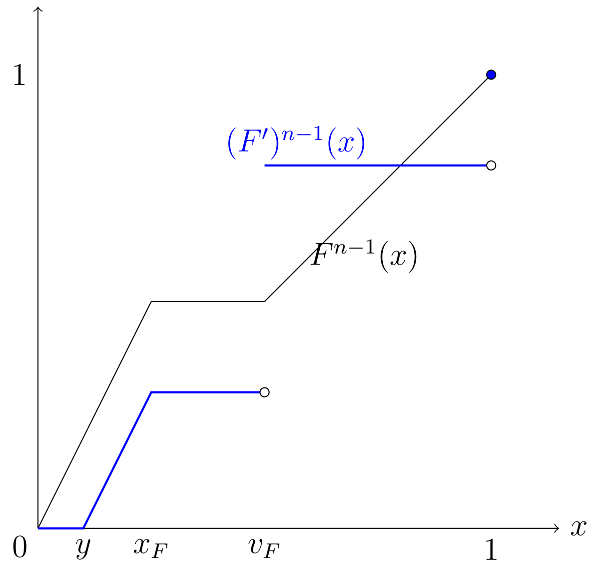

Figure 5: F^{n-1} and $(F')^{n-1}$ in Step 5 of the proof (for illustration purposes only).

We claim that

$$v_{F'} = \frac{p - c - \frac{\hat{q}}{n}(x_F - y)^{\frac{n}{n-1}} - \hat{q}y(x_F - y)^{\frac{1}{n-1}}}{1 - \hat{q}(x_F - y)^{\frac{1}{n-1}}}$$

for $y > 0$ sufficiently small. This is because (1) $v_{F'} < v_F$ and $v_{F'} \to v_F$ as $y \to 0$,[14] and (2) $v_{F'}$ satisfies that

$$\kappa \cdot (v_F - v_{F'}) + \left(1 - \hat{q}(x_F - y)^{\frac{1}{n-1}} - \kappa\right) \cdot (1 - v_{F'}) = c.$$

Pick y sufficiently small such that $0 < y < x_F < v_{F'} < v_F$. Thus, both F and F' have zero measure on $[v_{F'}, v_F)$, and sender 1's expected payoff by using F' when all the

[14]To see this, note that

$$v_{F'} = \frac{p - c - \frac{\hat{q}}{n}(x_F - y)^{\frac{n}{n-1}} - \hat{q}y(x_F - y)^{\frac{1}{n-1}}}{1 - \hat{q}(x_F - y)^{\frac{1}{n-1}}} < \frac{p - c - \frac{\hat{q}}{n}x_F^{\frac{n}{n-1}}}{1 - \hat{q}x_F^{\frac{1}{n-1}}} = v_F$$

for $y > 0$ sufficiently small, where the inequality is because the derivative of $v_{F'}$ (viewed as a function of y) is negative at $y = 0$, and the last equality follows from

$$\int_0^1 x \, \mathrm{d}F(x) = \int_0^{x_F} x \, \mathrm{d}F(x) + \int_{v_F}^1 x \, \mathrm{d}F(x) = \frac{\hat{q}}{n} x_F^{\frac{n}{n-1}} + v_F \cdot (1 - \hat{q}x_F^{\frac{1}{n-1}}) + c = p$$

(recall that F^{n-1} is linear on $[0, x_F]$ with slope q and flat on $[x_F, v_F)$).

other senders use F is

$$\begin{aligned}
&\int_y^{x_F} F^{n-1}(x)\,\mathrm{d}F'(x) + \big(1 - F'(v_F-)\big)\left[\frac{1}{n}\sum_{k=0}^{n-1} F^k(v_{F'})\right] \\
&= \frac{1}{n}\hat{q}^n (x_F - y)^{\frac{n}{n-1}} + \hat{q}^n y (x_F - y)^{\frac{1}{n-1}} + (1 - \hat{q}(x_F - y)^{\frac{1}{n-1}})\left[\frac{1}{n}\sum_{k=0}^{n-1} \hat{q}^k x_F^{\frac{k}{n-1}}\right] \\
&= \frac{1}{n} + \frac{n-1}{n}\hat{q}^n y (x_F - y)^{\frac{1}{n-1}} + \frac{1}{n}\hat{q}(x_F^{\frac{1}{n-1}} - (x_F - y)^{\frac{1}{n-1}})\sum_{k=0}^{n-2} \hat{q}^k x_F^{\frac{k}{n-1}} \\
&> \frac{1}{n}.
\end{aligned}$$

Thus, sender 1 has a profitable deviation. This completes the analysis of Case I.

Case II: $v_F = p - c$**.** We now consider the case in which $v_F = p - c$.

Step 6. If $(F, F, \ldots, F)$ *is a symmetric equilibrium, then*

$$1 + \sum_{k=1}^{n-1} \Pi_{1 \le j \le k} F(\psi_{n-j+1}(x)) \le \frac{x}{p-c}$$

for all $x \in [p - c, 1 - \frac{c}{p})$*.*

Suppose to the contrary, there is some $x^* \in [p - c, 1 - \frac{c}{p})$ such that

$$1 + \sum_{k=1}^{n-1} \Pi_{1 \le j \le k} F(\psi_{n-j+1}(x^*)) > \frac{x^*}{p-c}.$$

Since F is right continuous, there exists some $y \in [x^*, 1 - \frac{c}{p})$ such that F is continuous at $\{\psi_2(y), \psi_3(y), \ldots, \psi_n(y)\}$ and

$$1 + \sum_{k=1}^{n-1} \Pi_{1 \le j \le k} F(\psi_{n-j+1}(y)) > \frac{y}{p-c}.$$

Let

$$F_y(x) = \begin{cases} 1 - \frac{c}{1-y} - \frac{1}{y}(p - \frac{c}{1-y}) & \text{if } x \in [0, y); \\ 1 - \frac{c}{1-y} & \text{if } x \in [y, 1); \\ 1 & \text{if } x = 1. \end{cases}$$

Clearly, $F_y \in \mathcal{F}$ and $v_{F_y} = y$. Sender 1's payoff by using the strategy F_y when the other senders use the strategy F is[15]

$$\left[\frac{c}{1-y} + \frac{1}{y}(p - \frac{c}{1-y})\right]\left[\frac{1}{n} + \frac{1}{n}\sum_{k=1}^{n-1}\Pi_{1\le j\le k}F(\psi_{n-j+1}(y))\right] > \left[\frac{p}{y} - \frac{c}{y}\right]\frac{y}{n(p-c)} = \frac{1}{n}.$$

Thus, sender 1 has a profitable deviation. We have a contradiction.

Step 7. If

$$1 + \sum_{k=1}^{n-1} F^k(x) \le \frac{x}{p-c}$$

for all $x \in [p-c, 1-\frac{c}{p})$, then $(F, F, \ldots, F)$ is a symmetric equilibrium.

We show that none of the senders has a profitable deviation. By symmetry, we only show this for sender 1. Since $v_F = p - c$, we have $F(v_F-) = 0$. If sender 1 deviates to some F' with $v_{F'} \in [p-c, 1-\frac{c}{p})$, then his payoff is at most

$$\begin{aligned} &\int_{v_F}^{v_{F'}-}\frac{1}{n}\sum_{k=0}^{n-1}F^k(x)\,\mathrm{d}F'(x) + \int_{v_{F'}}^{1}\frac{1}{n}\sum_{k=0}^{n-1}F^k(v_{F'})\,\mathrm{d}F'(x) \\ \le &\int_{v_F}^{v_{F'}-}\frac{x}{n(p-c)}\,\mathrm{d}F'(x) + \int_{v_{F'}}^{1}\frac{v_{F'}}{n(p-c)}\,\mathrm{d}F'(x) \\ \le &\frac{1}{n(p-c)}\left[p - \int_{v_{F'}}^{1} x\,\mathrm{d}F'(x) + \int_{v_{F'}}^{1} v_{F'}\,\mathrm{d}F'(x)\right] \\ = &\frac{1}{n}, \end{aligned}$$

since for both terms in the first line we relax the calculations in two aspects: (a) the

[15] There are n cases in which the receiver visits sender 1. In the k-th case ($1 \le k \le n$), the receiver visits $k-1$ other senders before visiting sender 1. The probability of the first case is $\frac{1}{n}$. The probability of the $(k+1)$-th case taking into account the optimal search behavior of the receiver is $\frac{1}{n}\Pi_{1\le j\le k}F(\psi_{n-j+1}(y))$ (see Appendix A).

receiver always continues searching if sender 1 has not been visited and the maximum sampled reward so far is weakly less than $v_{F'}$; (b) the receiver always invests in sender 1's project whenever there is a tie. If sender 1 deviates to F'' with $v_{F''} = 1 - \frac{c}{p}$ (that is, $F'' = F_F$), then sender 1's payoff is at most

$$p\left[\frac{1}{n}\sum_{k=0}^{n-1} F^k(v_{F''})\right] \le p\frac{v_{F''}}{n(p-c)} = \frac{1}{n}.$$

Thus, sender 1 does not have a profitable deviation.

C Further discussions on equilibrium existence

Suppose that there are n senders. We study whether there is a feasible F that satisfy the sufficient condition for $(F, F, \ldots, F)$ to be a Nash equilibrium. That is, we ask if there is a feasible F such that

$$1 + \sum_{k=1}^{n-1} F^k(x) \le \frac{x}{p-c}$$

for all $x \in [p-c, 1-\frac{c}{p})$. We note that any such F satisfies that $v_F = p - c$, since F puts no measure on $[0, p-c)$.

Let

$$\tilde{x} = \min\{n \cdot (p-c), 1 - \frac{c}{p}\}.$$

Fix some $x \in [p-c, \tilde{x})$. Clearly, $1 \le \frac{x}{p-c} < n$. Consider

$$1 + \sum_{k=1}^{n-1} y^k = \frac{x}{p-c}.$$

Note that the left hand side is strictly increasing in y for $y \in [0,1]$, is 1 when $y = 0$ and n when $y = 1$. Thus, there is a unique solution within $[0,1]$ such that the equation holds. Let $y_n(x)$ denote this unique solution.

It is clear that $y_n(x)$ is increasing in x, and decreasing in n. In general, it would be tedious to specify the solution $y_n(x)$, if possible at all. We extend the definition of $y_n(x)$ by letting it be 0 for $x \in [0, p-c)$, and 1 for $x \in [\tilde{x}, 1]$. Then $y_n(x)$ is a distribution. If

$$\int_{p-c}^{1} x \, \mathrm{d}y_n(x) \leq p,$$

then a feasible F with the mean p exists, which satisfies the condition that for any $x \in [p-c, 1-\frac{c}{p})$,

$$1 + \sum_{k=1}^{n-1} F^k(x) \leq \frac{x}{p-c}.$$

We further note that when n is large, $\tilde{x} = 1 - \frac{c}{p}$. As $y_n(x)$ is decreasing in n, for any $x \in [p-c, 1-\frac{c}{p})$, $y_n(x) \to 0$ as $n \to \infty$. That is, $y_n(x)$ will put more probability above $1 - \frac{c}{p}$, implying that it is more difficult to satisfy the mean constraint.

D Proof of Theorem 2

Clearly, in any symmetric equilibrium, all senders get the same expected payoff of $\frac{1}{n}$.

We show that there is no symmetric equilibrium in the case in which $p - c < v_F \leq v_G$. Suppose to the contrary, there exists such a symmetric equilibrium $(F, F, \ldots, F)$. Step 1 - Step 4 below establish properties that F necessarily satisfies. Step 5 then shows that sender 1 has a profitable deviation, which contradicts that $(F, F, \ldots, F)$ is a Nash equilibrium.

Step 1. Since $v_F > p - c$, *we have* $F(v_F-) \neq 0$.

Step 2. Since H *is atomless and* F *is an MPC of* H, *we have* $F(0) = 0$.

Step 3. F *has no jumps on* $(0, v_F)$.

Suppose that F has a jump at some $z \in (0, v_F)$. We show that sender 1 has a profitable deviation, which contradicts that $(F, F, \ldots, F)$ is a Nash equilibrium. Let $F = E_1 + E_2$, where

1. E_1 is the restriction of F on $[0, z) \cup (z, 1]$ (that is, E_1 is a finite measure with total measure $1 - F(\{z\})$ such that $E_1(A) = F(A)$ for any $A \subseteq [0, z) \cup (z, 1]$), and

2. E_2 is the restriction of F on $\{z\}$.

Since F is an MPC of H, H can be decomposed as $H = H_1 + H_2$ such that E_j is an MPC of H_j for $j = 1, 2$. Note that

$$\int_{[0,1]} x \,\mathrm{d}H_2(x) = \int_{[0,1]} x \,\mathrm{d}E_2(x) = z \cdot F(\{z\}), \text{ and } H_2([0,1]) = E_2([0,1]) = F(\{z\}).$$

Since E_2 is an MPC of H_2 and H_2 is atomless (since H is atomless), $H_2([0, z)) > 0$. For any $0 < \epsilon < H_2([0, z))$, let $x_\epsilon := \sup\{x : H_2([0, x_\epsilon]) \leq \epsilon\}$. Let E_2^ϵ be such that

1. $E_2^\epsilon(A) = H_2(A)$ for any $A \subseteq [0, x_\epsilon]$, and
2. On the interval $(x_\epsilon, 1]$, E_2^ϵ concentrates on the point $z^\epsilon = \frac{1}{H_2((x_\epsilon,1])} \int_{(x_\epsilon,1]} x \,\mathrm{d}H_2(x)$ with measure $H_2((x_\epsilon, 1])$.

By construction, $z^\epsilon > z$. Furthermore, as $\epsilon \to 0$,

$$H_2((\epsilon, 1]) \to H_2([0, 1]) = F(\{z\}),\ z^\epsilon \to z, \text{ and } z^\epsilon < v_F.$$

Define a probability measure $F^\epsilon := E_1 + E_2^\epsilon$. By construction, F^ϵ is an MPC of H and has the same reservation value as F. Sender 1's expected payoff from F^ϵ while all the other senders use F converges to $\tilde{v}$ as $\epsilon \to 0$, where $\tilde{v}$ is sender 1's expected payoff in the following hypothetical scenario:

- All senders choose the same strategy F. The receiver adopts a tie-breaking rule that treats all senders identically except when the posterior for sender 1 is z, in which case the receiver always chooses sender 1 when resolving the tie.

Since $F(\{z\}) > 0$, we have $\tilde{v} > \frac{1}{n}$. Thus, sender 1 has a profitable deviation.

Step 4. F^{n-1} is flat on $[x_F, v_F)$ for some $0 < x_F < v_F$.

Suppose to the contrary, F^{n-1} is not flat on $[x, v_F)$ for any $x < v_F$. For some $z < v_F$ that is sufficiently close to v_F, let y_z be such that

$$\int_z^{y_z} x \,\mathrm{d}F(x) = v_F \cdot F([z, y_z]).$$

Clearly, $y_z > v_F$. Consider an MPC of F, denoted F_z, as follows.

1. F_z and F coincide on $[0, z) \cup (y_z, 1]$, and
2. F_z concentrates at v_F with the finite measure $F([z, y_z])$.

Since F_z is an MPC of F, $v_{F_z} \leq v_F$. We claim that $v_{F_z} > z$. If $v_{F_z} \leq z$, then

$$\begin{aligned}
&\int_{[v_F,1]} (x - v_F)\,\mathrm{d}F(x) = c = \int_{[v_{F_z},1]} (x - v_{F_z})\,\mathrm{d}F_z(x) \geq \int_{[z,1]} (x - v_{F_z})\,\mathrm{d}F_z(x) \\
&= \int_{[z,v_F)} x\,\mathrm{d}F(x) + \int_{[v_F,1]} x\,\mathrm{d}F(x) - v_{F_z} \cdot F_z([z,1]) \\
&> v_{F_z} \cdot F([z, v_F)) + \int_{[v_F,1]} x\,\mathrm{d}F(x) - v_{F_z} \cdot F_z([z,1]) \\
&> \int_{[v_F,1]} x\,\mathrm{d}F(x) - v_{F_z} \cdot F([v_F, 1]),
\end{aligned}$$

implying that $v_{F_z} > v_F$, which contradicts with $v_{F_z} \leq v_F$. Thus, $v_{F_z} > z$.

By adopting F_z when all the other senders choose F, sender 1's expected payoff is at least

$$\begin{aligned}
&\int_{[0,z)} F^{n-1}(x)\,\mathrm{d}F(x) + (1 - F([0,z)))\left[\frac{1}{n}\sum_{k=0}^{n-1} F^k(v_{F_z})\right] \\
&> \int_{[0,z)} F^{n-1}(x)\,\mathrm{d}F(x) + (1 - F([0,z)))\left[\frac{1}{n}\sum_{k=0}^{n-1} F^k(z)\right] \\
&= \frac{1}{n}F^n(z) + (1 - F([0,z)))\frac{1}{n}\frac{1 - F^n(z)}{1 - F(z)} \\
&= \frac{1}{n}.
\end{aligned}$$

Thus, sender 1 has a profitable deviation, which contradicts that $(F, F, \ldots, F)$ is a Nash equilibrium.

Step 5. Sender 1 *has a profitable deviation.*

Consider an MPC of F, denoted F', as follows.

$$F'(x) = \begin{cases} 0, & \text{if } x \in [0, y); \\ F(x) - F(y), & \text{if } x \in [y, v_F); \\ F(\overline{v}), & \text{if } x \in [v_F, \overline{v}); \\ F(x), & \text{if } x \in [\overline{v}, 1]; \end{cases}$$

where $F(y) > 0$ is sufficiently small and $\bar{v}$ is such that

$$v_F \cdot (F(\overline{v}) - (F(v_F) - F(y))) = \int_{[0,y)} x \, \mathrm{d}F(x) + \int_{[v_F,\overline{v})} x \, \mathrm{d}F(x).$$

Since F' is an MPC of F, we have $v_{F'} < v_F$. Furthermore, $v_{F'} \to v_F$ as $y \to 0$. Thus, for sufficiently small $y > 0$, $v_{F'} > x_F$, and F has no measure on the interval $[v_{F'}, v_F)$.

Sender 1's expected payoff by using F' when all the other senders use F is

$$\begin{aligned}
&\int_y^{v_{F'}} F^{n-1}(x) \, \mathrm{d}F'(x) + \big(1 - F'(v_F-)\big) \left[\frac{1}{n} \sum_{k=0}^{n-1} F^k(v_F-)\right] \\
&= \int_y^{v_{F'}} F^{n-1}(x) \, \mathrm{d}F(x) + \big(1 - F(v_F-) + F(y)\big) \left[\frac{1}{n} \sum_{k=0}^{n-1} F^k(v_F-)\right] \\
&= \int_0^{v_F-} F^{n-1}(x) \, \mathrm{d}F(x) - \int_0^{y} F^{n-1}(x) \, \mathrm{d}F(x) + \big(1 - F(v_F-) + F(y)\big) \left[\frac{1}{n} \sum_{k=0}^{n-1} F^k(v_F-)\right] \\
&= \frac{1}{n} + F(y) \left[\frac{1}{n} \sum_{k=0}^{n-1} F^k(v_F-)\right] - \int_0^{y} F^{n-1}(x) \, \mathrm{d}F(x) \\
&> \frac{1}{n}.
\end{aligned}$$

This completes the proof of Theorem 2.

E Proof of Theorem 3

We classify our analysis into three cases. We first show that there is no Nash equilibrium when $v_{F_1} < v_{F_2}$ (**Case I**) or when $v_{F_1} = v_{F_2}$ (**Case II**). We then consider **Case III** in which $v_{F_1} > v_{F_2}$.

Case I. Suppose that there exists a Nash equilibrium (F_1, F_2) such that $v_{F_1} < v_{F_2}$.

Step 1. $F_1((x, v_{F_2})) > 0$ for any $x \in (v_{F_1}, v_{F_2})$.

Suppose to the contrary, there exists some $z \in (v_{F_1}, v_{F_2})$ such that $F_1(z, v_{F_2}) = 0$. We claim that sender 2 has a profitable deviation. Indeed, we construct the strategy F_2' by performing a MPC of F_2 that shifts a sufficiently small probability on $[0, v_{F_1})$ and $(v_{F_2}, 1]$ to the point $\{v_{F_2}\}$. Clearly, $z < v_{F_2'} < v_{F_2}$, and sender 2 obtains a higher expected payoff by using F_2'.[16]

Step 2. F_1 has no jump on (v_{F_1}, v_{F_2}).

Suppose to the contrary, $F_1(\{z\}) > 0$ for some $z \in (v_{F_1}, v_{F_2})$.

1. It must be that $F_2(\{z\}) = 0$, as otherwise sender 2 could benefit by performing a mean-preserving spread at the posterior z.
2. Furthermore, if $F_2((a, b)) = 0$ for some open neighborhood (a, b) of z, then sender 1 would like to split the mass at z to $\{\frac{a+z}{2}\}$ and $\{v_{F_2}\}$ so that he could obtain a higher payoff. Thus, $F_2((a, b)) > 0$ for arbitrarily small open neighborhood (a, b) of z.
3. But then sender 2 could obtain a higher payoff by choosing a mean-preserving contraction on (a, b) so that the concentration point is $z + \epsilon$.

Step 3. F_2 has no jump on (v_{F_1}, v_{F_2}).

Suppose to the contrary, $F_2(\{z\}) > 0$ for some $z \in (v_{F_1}, v_{F_2})$.

1. It must be that $F_1(\{z\}) = 0$, as otherwise sender 1 could benefit by performing a mean-preserving spread at the posterior z.

[16] Note that

$$v_{F_2} > v_{F_1} \geq p_1 - c > p_2 - c \Longrightarrow F_2([0, v_{F_1})) > 0 \text{ and } F_2((v_{F_2}, 1]) > 0.$$

2. Furthermore, if $F_1((a,b)) = 0$ for some open neighborhood (a,b) of z, then sender 2 would like to split the mass at z to $\{\frac{a+z}{2}\}$ and $\{v_{F_2}\}$ so that he could obtain a higher payoff. Note that sender 2's payoffs at z and $\frac{a+z}{2}$ are the same, while the payoff at v_{F_2} is higher than the payoff at z since $F_1(a, v_{F_2}) > 0$ for any $a \in (v_{F_1}, v_{F_2})$. Thus, $F_1((a,b)) > 0$ for arbitrarily small open neighborhood (a,b) of z.
3. But then sender 1 could obtain a higher payoff by choosing a mean-preserving contraction on (a,b) so that the concentration point is $z + \epsilon$.

Step 4. On (v_{F_1}, v_{F_2}), F_1 and F_2 have the same support.

If there exists $(a,b) \cap \text{supp}(F_2) = \emptyset$ but $F_1(a,b) > 0$, then sender 1 can strictly benefit by a mean-preserving spread, which puts all the measure on (a,b) on the two points a and v_{F_2}.

If there exists $(a,b) \cap \text{supp}(F_1) = \emptyset$ but $F_2(a,b) > 0$, then sender 2 can strictly benefit by a mean-preserving spread, which puts all the measure on (a,b) on the two points a and v_{F_2}.

Step 5. On (v_{F_1}, v_{F_2}), both F_1 and F_2 are linear.

If not, some sender could do a mean-preserving spread or a mean-preserving contraction to obtain a higher payoff.

Step 6. Sender 1 has a profitable deviation.

Suppose that $F_1(x) = a_1 x + b_1$ and $F_2(x) = a_2 x + b_2$ for $x \in (v_{F_1}, v_{F_2})$. For sender 1, we consider a mean-preserving spread on $(v_{F_2} - \epsilon, v_{F_2})$ by concentrating the measure on $v_{F_2} - \epsilon$ and v_{F_2}.

The expected payoff of sender 1 based on this part of measure before the spread is

$$
\begin{aligned}
&\int_{v_{F_2}-\epsilon}^{v_{F_2}} \left(\frac{1}{2}F_2(x) + \frac{1}{2}F_2(v_{F_1}-)\right) \mathrm{d}F_1(x) \\
=\ &\frac{1}{2}F_1(v_{F_2}-\epsilon, v_{F_2}) \cdot \left[\left(\frac{1}{2}F_2(v_{F_2}-\epsilon) + \frac{1}{2}F_2(v_{F_1}-)\right) + \left(\frac{1}{2}F_2(v_{F_2}-) + \frac{1}{2}F_2(v_{F_1}-)\right)\right].
\end{aligned}
$$

The expected payoff of sender 1 based on this part of measure after the spread is

$$\frac{1}{2}F_1(v_{F_2}-\epsilon, v_{F_2})\cdot\left[\left(\frac{1}{2}F_2(v_{F_2}-\epsilon)+\frac{1}{2}F_2(v_{F_1}-)\right)+\left(\frac{1}{2}+\frac{1}{2}F_2(v_{F_1}-)\right)\right]$$
$$>\frac{1}{2}F_1(v_{F_2}-\epsilon, v_{F_2})\cdot\left[\left(\frac{1}{2}F_2(v_{F_2}-\epsilon)+\frac{1}{2}F_2(v_{F_1}-)\right)+\left(\frac{1}{2}F_2(v_{F_2}-)+\frac{1}{2}F_2(v_{F_1}-)\right)\right].$$

The payoff of sender 1 at any other posterior is unchanged. Thus, sender 1 has a profitable deviation, which contradicts that (F_1, F_2) is a Nash equilibrium.

We conclude that there is no equilibrium such that $v_{F_1} < v_{F_2}$. This completes the analysis of **Case I**.

Case II. Adopting similar arguments as in **Case I**, we can show that there is no equilibrium such that $v_{F_1} = v_{F_2}$. We omit the details.

Case III. Lastly, we consider the case in which $v_{F_1} > v_{F_2}$.

Step 7. Either $F_2([v_{F_2}, v_{F_1})) = 0$ *or* $F_2((v_{F_1}, 1]) = 0$.

Otherwise, Sender 2 could achieve a higher expected payoff by choosing a MPC that shifts some measure from $[v_{F_2}, v_{F_1})$ and $(v_{F_1}, 1]$ to v_{F_1} without changing the reservation value.

Step 8. $F_2([v_{F_2}, v_{F_1})) = 0$.

It suffices to show that $F_2((v_{F_1}, 1]) > 0$. Suppose instead we have $F_2((v_{F_1}, 1]) = 0$.

1. There are two cases to consider. In the first case, suppose that $F_2((v_{F_2}, v_{F_1})) = 0$. In other words, on the interval $[v_{F_2}, 1]$, F_2 concentrates on the two points v_{F_2} and v_{F_1} such that $F_2(\{v_{F_1}\}) > 0$. When $v_{F_1} < 1 - \frac{c}{p_1}$, sender 1 could do a mean-preserving spread, which splits a sufficiently small measure on $(v_{F_1}, 1]$ to the two end points 0 and 1. The new reservation value is slightly higher than v_{F_1}, and sender 1 obtains a higher payoff, a contradiction. When $v_{F_1} = 1 - \frac{c}{p_1}$, then sender 1 can benefit by a mean-preserving contraction, which moves a sufficiently small measure from 0 and 1 to v_{F_1}, a contradiction.
2. In the second case, suppose that $F_2((v_{F_2}, v_{F_1})) > 0$. If $F_1((v_{F_2}, v_{F_1})) = 0$, then sender 2 can benefit by splitting the measure on $(v_{F_2} + \epsilon, v_{F_1})$ into the two points

$v_{F_2} + \epsilon$ and v_{F_1} for sufficiently small ϵ, a contradiction. Thus, $F_1((v_{F_2}, v_{F_1})) > 0$. Following the same arguments as in **Case I**, F_1 and F_2 are linear on (v_{F_2}, v_{F_1}), and then sender 2 has a profitable deviation, a contradiction.

Step 9. $F_1([0, v_{F_2})) = 0$.

Suppose to the contrary, $F_1([0, v_{F_2})) > 0$. Then sender 1 can benefit from a mean-preserving contraction, which moves some measures on $[0, v_{F_2})$ and $(v_{F_1}, 1]$ to v_{F_1}, a contradiction.

Step 10. $v_{F_2} = p_2 - c$.

If $v_{F_2} > p_2 - c$, then $F_2([0, v_{F_2})) > 0$. Sender 2 can strictly benefit from a mean-preserving contraction, which moves some measures on $[0, v_{F_2})$ and $(v_{F_1}, 1]$ to v_{F_1}, a contradiction.

From Step 9, we have that F_1 places no probability on the interval $[0, v_{F_2})$. From Step 8 and Step 10, we have that $v_{F_2} = p_2 - c$, and F_2 places no probability on the interval $[0, v_{F_1})$. Thus, in any equilibrium (F_1, F_2), the receiver meets each sender i with equal probability $\frac{1}{2}$, and invests in his project regardless of the posterior q_i. The payoff of each sender is $\frac{1}{2}$, and the receiver's expected payoff is $\frac{p_1+p_2}{2}$.

Step 11. If (F_1, F_2) is a Nash equilibrium, then $1 + F_2(x) \leq \frac{x}{p_1 - c}$ for all $x \in [p_1 - c, 1 - \frac{c}{p_1})$, $2F_1(x) \leq \frac{x}{p_2 - c}$ for all $x \in [p_2 - c, v_{F_1})$, and $1 + F_1(x) \leq \frac{x}{p_2 - c}$ for all $x \in [v_{F_1}, 1 - \frac{c}{p_2})$.

Suppose that for some $x^* \in [p_1 - c, 1 - \frac{c}{p_1})$, we have $1 + F_2(x^*) > \frac{x^*}{p_1 - c}$. Since F_2 is right continuous, there exists some $y \in [x^*, 1 - \frac{c}{p_1})$ such that F_2 is continuous at y and $1 + F_2(y) > \frac{y}{p_1 - c}$. Let

$$F_1'(x) = \begin{cases} 1 - \frac{c}{1-y} - \frac{1}{y}(p_1 - \frac{c}{1-y}) & \text{if } x \in [0, y); \\ 1 - \frac{c}{1-y} & \text{if } x \in [y, 1); \\ 1 & \text{if } x = 1. \end{cases}$$

Clearly, the mean of F_1' is p_1 and $v_{F_1'} = y$. Sender 1's payoff by using the strategy F_1'

when sender 2 uses the strategy F_2 is

$$\left[\frac{c}{1-y} + \frac{1}{y}(p_1 - \frac{c}{1-y})\right]\left[\frac{1}{2} + \frac{1}{2}F_2(y)\right] > \left[\frac{p_1}{y} - \frac{c}{y}\right]\frac{y}{2(p_1-c)} = \frac{1}{2}.$$

Thus, sender 1 has a profitable deviation, and (F_1, F_2) is not a Nash equilibrium. We have a contradiction.

Suppose that for some $x^* \in [v_{F_1}, 1 - \frac{c}{p_2})$, we have $1 + F_1(x^*) > \frac{x^*}{p_2 - c}$. Since F_1 is right continuous, there exists some $y \in [x^*, 1 - \frac{c}{p_2})$ such that F_1 is continuous at y and $1 + F_1(y) > \frac{y}{p_2 - c}$. Let

$$F_2'(x) = \begin{cases} 1 - \frac{c}{1-y} - \frac{1}{y}(p_2 - \frac{c}{1-y}) & \text{if } x \in [0, y); \\ 1 - \frac{c}{1-y} & \text{if } x \in [y, 1); \\ 1 & \text{if } x = 1. \end{cases}$$

Sender 2's payoff by using the strategy F_2' when sender 1 uses the strategy F_1 is

$$\left[\frac{c}{1-y} + \frac{1}{y}(p_2 - \frac{c}{1-y})\right]\left[\frac{1}{2} + \frac{1}{2}F_1(y)\right] > \left[\frac{p_2}{y} - \frac{c}{y}\right]\frac{y}{2(p_2-c)} = \frac{1}{2}.$$

Thus, sender 2 has a profitable deviation, and (F_1, F_2) is not a Nash equilibrium. We have a contradiction.

Suppose that for some $x^* \in [p_2 - c, v_{F_1})$, we have $2F_1(x^*) > \frac{x^*}{p_2 - c}$. Since F_1 is right continuous, there exists some $y \in [x^*, v_{F_1})$ such that F_1 is continuous at y and $2F_1(y) > \frac{y}{p_2 - c}$. Let

$$F_2''(x) = \begin{cases} 1 - \frac{c}{1-y} - \frac{1}{y}(p_2 - \frac{c}{1-y}) & \text{if } x \in [0, y); \\ 1 - \frac{c}{1-y} & \text{if } x \in [y, 1); \\ 1 & \text{if } x = 1. \end{cases}$$

Sender 2's payoff by using the strategy F_2'' when sender 1 uses the strategy F_1 is

$$\begin{aligned}
&\frac{c}{1-y}\left[\frac{1}{2}+\frac{1}{2}F_1(y)\right]+\frac{1}{y}(p_2-\frac{c}{1-y})F_1(y)\\
\geq&\left[\frac{c}{1-y}+\frac{1}{y}(p_2-\frac{c}{1-y})\right]F_1(y)\\
>&\left[\frac{p_2}{y}-\frac{c}{y}\right]\frac{y}{2(p_2-c)}\\
=&\frac{1}{2}.
\end{aligned}$$

Thus, sender 2 has a profitable deviation, and (F_1, F_2) is not a Nash equilibrium. We have a contradiction.

Step 12. If $1+F_2(x) \leq \frac{x}{p_1-c}$ *for all* $x \in [p_1-c, 1-\frac{c}{p_1})$ *and* $1+F_1(x) \leq \frac{x}{p_2-c}$ *for all* $x \in [p_2-c, 1-\frac{c}{p_2})$*, then* (F_1, F_2) *is a Nash equilibrium.*

We show that neither sender has a profitable deviation. First consider sender 1. If sender 1 deviates to some F_1' with $v_{F_1'} \in [p_1-c, 1-\frac{c}{p_1})$, then his payoff is

$$\begin{aligned}
&\int_{v_{F_2}}^{v_{F_1'}-}\left[\frac{1}{2}+\frac{1}{2}\left[F_2(x-)+\frac{1}{2}F_2(\{x\})\right]\right]\mathrm{d}F_1'(x)+\int_{v_{F_1'}}^{1}\left[\frac{1}{2}+\frac{1}{2}F_2(v_{F_1'}-)\right]\mathrm{d}F_1'(x)\\
\leq&\int_{v_{F_2}}^{v_{F_1'}-}\frac{x}{2(p_1-c)}\,\mathrm{d}F_1'(x)+\int_{v_{F_1'}}^{1}\frac{v_{F_1'}}{2(p_1-c)}\,\mathrm{d}F_1'(x)\\
\leq&\frac{1}{2(p_1-c)}\left[p_1-\int_{v_{F_1'}}^{1}x\,\mathrm{d}F_1'(x)+\int_{v_{F_1'}}^{1}v_{F_1'}\,\mathrm{d}F_1'(x)\right]\\
=&\frac{1}{2},
\end{aligned}$$

where the last line uses the definition of the reservation value. If sender 1 deviates to F_1' with $v_{F_1'} = 1-\frac{c}{p_1}$, then his payoff is

$$p_1\left[\frac{1}{2}+\frac{1}{2}F_2(v_{F_1'}-)\right] \leq p_1\frac{v_{F_1'}}{2(p_1-c)}=\frac{1}{2}.$$

Thus, sender 1 does not have a profitable deviation.

Next, we consider sender 2. If sender 2 deviates to some F_2' with $v_{F_2'} \in [p_2-c, v_{F_1})$, then his payoff is

$$
\begin{aligned}
&\int_{v_{F_2}}^{v_{F_2'}-} \left[F_1(x-) + \frac{1}{2}F_1(\{x\})\right] \mathrm{d}F_2'(x) + \int_{v_{F_2'}}^{v_{F_1}-} \left[\frac{1}{2}\left[F_1(x-) + \frac{1}{2}F_1(\{x\})\right] + \frac{1}{2}F_1(v_{F_2'}-)\right] \mathrm{d}F_2'(x) \\
&+ \int_{v_{F_1}}^{1} \left[\frac{1}{2} + \frac{1}{2}F_1(v_{F_2'}-)\right] \mathrm{d}F_2'(x) \\
&\leq \int_{v_{F_2}}^{v_{F_2'}-} \frac{x}{2(p_2-c)} \mathrm{d}F_2'(x) + \int_{v_{F_2'}}^{v_{F_1}-} \frac{x}{2(p_2-c)} \mathrm{d}F_2'(x) + \int_{v_{F_1}}^{1} \frac{v_{F_2'}}{2(p_2-c)} \mathrm{d}F_2'(x) \\
&\leq \frac{1}{2(p_1-c)}\left[p_2 - \int_{v_{F_1}}^{1} x \,\mathrm{d}F_2'(x) + \int_{v_{F_1}}^{1} v_{F_2'} \,\mathrm{d}F_2'(x)\right] \\
&= \frac{1}{2},
\end{aligned}
$$

where the last line uses the definition of the reservation value. If sender 2 deviates to some F_2' with $v_{F_2'} \in [v_{F_1}, 1-\frac{c}{p_2})$, then his payoff is

$$
\begin{aligned}
&\int_{v_{F_2}}^{v_{F_1}-} \left[F_1(x-) + \frac{1}{2}F_1(\{x\})\right] \mathrm{d}F_2'(x) + \int_{v_{F_1}}^{1} \left[\frac{1}{2} + \frac{1}{2}F_1(v_{F_2'}-)\right] \mathrm{d}F_2'(x) \\
&\leq \int_{v_{F_2}}^{v_{F_1}-} \frac{x}{2(p_1-c)} \mathrm{d}F_2'(x) + \int_{v_{F_1}}^{1} \frac{v_{F_2'}}{2(p_1-c)} \mathrm{d}F_2'(x) \\
&\leq \frac{1}{2(p_1-c)}\left[p_1 - \int_{v_{F_1}}^{1} x \,\mathrm{d}F_2'(x) + \int_{v_{F_1}}^{1} v_{F_2'} \,\mathrm{d}F_2'(x)\right] \\
&= \frac{1}{2}.
\end{aligned}
$$

If sender 2 deviates to F_2' with $v_{F_2'} = 1-\frac{c}{p_2}$, then his payoff is

$$
p_2\left[\frac{1}{2} + \frac{1}{2}F_1(v_{F_2'}-)\right] \leq p_2\frac{v_{F_2'}}{2(p_2-c)} = \frac{1}{2}.
$$

Thus, sender 2 does not have a profitable deviation.

This completes the proof of Theorem 3.